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Performance optimization for two-stage thermoelectric refrigerator system driven by two-stage thermoelectric generator

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ABSTRACT

A new configuration of combined thermoelectric device, two-stage thermoelectric refrigerator driven by two-stage thermoelectric generator, is proposed in this paper. The thermodynamic model of the combined device is built by using non-equilibrium thermodynamic theory. The analytical formulae for the stable working electrical current, the cooling load versus the working electrical current, and the coefficient of performance (COP) versus the working electrical current of the combined device are derived. For the fixed total number of thermoelectric elements of the combined device, the allocations of the thermoelectric element pairs among the two thermoelectric generators and the two thermoelectric refrigerators are optimized for maximum cooling load and COP, respectively. The influences of the heat source temperature of the two-stage thermoelectric generator and the heat source (cooling space) temperature of the two-stage thermoelectric refrigerator on the optimal performance of the combined thermoelectric device are analyzed by detailed numerical examples.

1. Introduction

Semiconductor thermoelectric power generation, based on the Seebeck effect, and semiconductor thermoelectric cooling, based on the Peltier effect, have very interesting capabilities compared to conventional power generation and cooling systems [1–4]. The absence of moving components results in an increase of reliability, a reduction of maintenance, and an increase of system life; the modularity allows for application in a wide-scale range without significant losses in performance; the absence of a working fluid avoids environmental dangerous leakage; and the noise reduction appears also to be an important feature. Thermoelectric generator and refrigerator have been used in military, aerospace, instrument, and industrial or commercial products, as a power generation and cooling devices for specific purposes. Many researchers are concerned about the physical properties of thermoelectric material and the manufacturing technique of thermoelectric modules [5–8]. In addition to the improvement of the thermoelectric material and module, the system analysis and optimization of thermoelectric generator and refrigerator are equally important in designing high-performance thermoelectric generators and refrigerators.

In general, conventional non-equilibrium thermodynamics [1,9,10] is used to analyze the performance of single-stage one- or multiple-element thermoelectric generators [11–20] and refrigerators [21–32]. Due to the performance limits of thermoelectric material, thermoelectric generators and refrigerators of two stages or more should be applied to improve the level of thermodynamic performance. The performance analysis and optimization of two-stage thermoelectric generators [33,34] and refrigerators [35–41] were also performed. All of those were performed by using conventional non-equilibrium thermodynamics without considering the external losses. Performances of two-stage thermoelectric generators [42] and refrigerators [43] with external heat transfer are analyzed using the combination of finite time thermodynamics and non-equilibrium thermodynamics by Chen et al. [42,43].

All objects of these researches are dependent thermoelectric devices, that is, they have a direct-current power source providing direct current to refrigerate or heat up. However, for some special systems, such as submarines, cars, and special electric circuit, the heat rejected from the thermal machine may drive a thermoelectric refrigerator through the use of a thermoelectric generator, so that the thermoelectric cooler does not need an independent power source. Such a new refrigeration system is directly composed of a thermoelectric generator and a thermoelectric cooler. It is different from the traditional thermoelectric systems which merely consist of thermoelectric generators or coolers, and deserves to be investigated both from the point of view of theoretical design, as well as practical application. Chen et al. [44] and Khattab and El Shenawy [45] built a model of this kind of combined system, single-stage thermoelectric refrigerator driven by single-stage thermoelectric generator, and analyzed the performance of the device. Based on the performance analysis and optimization of thermoelectric generator and heat pump by using non-equilibrium

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thermodynamics, Meng et al. [46] built a model of single-stage thermoelectric heat pump driven by single-stage thermoelectric generator, and analyzed and optimized its performance.

Based on the performance analysis and optimization of two-stage thermoelectric generators [33,34] and two-stage thermoelectric refrigerators [35–41] by using non-equilibrium thermodynamics, this paper provides a new configuration of combined thermoelectric device, two-stage thermoelectric refrigerator driven by two-stage thermoelectric generator. The thermodynamic model of the combined device is built by using non-equilibrium thermodynamic theory. Three analytical formulae for the stable working electrical current, the cooling load versus the working electrical current, and the coefficient of performance (COP) versus the working electrical current of the combined device are derived. For a fixed total number of thermoelectric elements of the combined device, the thermoelectric element allocations among the two thermoelectric generators and the two thermoelectric refrigerators are optimized for maximizing cooling load and COP, respectively. There has been no investigation concerning the performance analysis and optimization for such combined thermoelectric devices published in the open literature. The influences of the heat source temperature of the two-stage thermoelectric generator and the heat source (cooling space) temperature of the two-stage thermoelectric refrigerator on the optimal performance of the combined device are analyzed by detailed numerical examples.

2. Model of a combined thermoelectric device

A schematic diagram of a combined thermoelectric device is shown in Fig. 1. The device consists of a two-stage thermoelectric generator and a two-stage thermoelectric refrigerator in series. The direct-current power source of the refrigerator is the current of the generator.

The generator consists of a top stage with m_1 pairs of thermoelectric elements and a bottom stage with m_2 pairs of thermoelectric elements. The total number of thermoelectric element pairs of the generator is m, i.e. $m = m_1 + m_2$. Each element is composed of a P-type and a N-type semiconductor legs. The thermoelectric generating element is assumed to be insulated, both electrically and thermally, from its surroundings, except at the junction–reservoir contacts and the junction between the two stages. The internal irreversibility is caused by Joule loss and heat conduction loss through the semiconductor between the hot and cold junctions. The Joule loss generates an internal heat I^2R, where R is the total internal electrical resistance of the semiconductor couple and I is the working electrical current generating from the semiconductor couple. The conduction heat losses are $K(T_{H1} - T_m)$ for the top stage and $K(T_m - T_{L1})$ for the bottom stage, respectively, where K is the thermal conductance of the semiconductor couple, T_{H1} is the hot junction (heat source) temperature, T_{L1} is the cold junction (heat sink) temperature, and T_m is the temperature of the junction between the two stages. The rates of heat flow of the thermoelectric generator are Q_{H1}, Q_m, and Q_{L1}.

The refrigerator consists of a top stage with n_1 pairs of thermoelectric elements and a bottom stage with n_2 pairs of thermoelectric elements. The total number of thermoelectric element pairs of the refrigerator is n, i.e. $n = n_1 + n_2$. Each element is composed of a P-type and a N-type semiconductor legs. The structure of the refrigerator is similar to the generator. The conduction heat losses are $K(T_{H2} - T_n)$ for the top stage and $K(T_n - T_{L2})$ for the bottom stage, respectively, where T_{H2} is the hot junction (heat sink) temperature, T_{L2} is the cold junction (heat source) temperature, and T_n is the temperature of the junction between the two stages. The rates of heat flow of the thermoelectric refrigerator are Q_{H2}, Q_n, and Q_{L2}.

The total number of thermoelectric element pairs of the combined thermoelectric device, M, is finite and $M = m + n$ holds. The cooling load of the combined thermoelectric device is Q_{L2}. The power input required by the refrigerator is the power output of the generator.

According to the theory of non-equilibrium thermodynamics, for the two-stage thermoelectric generator, one has

$$Q_{H1} = m_1\left[\alpha I T_{H1} + K(T_{H1} - T_m) - \frac{1}{2}I^2R\right] \tag{1}$$

$$Q_m = m_1\left[\alpha I T_m + K(T_{H1} - T_m) + \frac{1}{2}I^2R\right] \tag{2}$$

$$Q_m = m_2\left[\alpha I T_m + K(T_m - T_{L1}) - \frac{1}{2}I^2R\right] \tag{3}$$

$$Q_{L1} = m_2\left[\alpha I T_{L1} + K(T_m - T_{L1}) + \frac{1}{2}I^2R\right] \tag{4}$$

For the two-stage thermoelectric refrigerator, one has

$$Q_{H2} = n_1\left[\alpha I T_{H2} - K(T_{H2} - T_n) + \frac{1}{2}I^2R\right] \tag{5}$$

$$Q_n = n_1\left[\alpha I T_n - K(T_{H2} - T_n) - \frac{1}{2}I^2R\right] \tag{6}$$

$$Q_n = n_2\left[\alpha I T_n - K(T_n - T_{L2}) + \frac{1}{2}I^2R\right] \tag{7}$$

$$Q_{L2} = n_2\left[\alpha I T_{L2} - K(T_n - T_{L2}) - \frac{1}{2}I^2R\right] \tag{8}$$

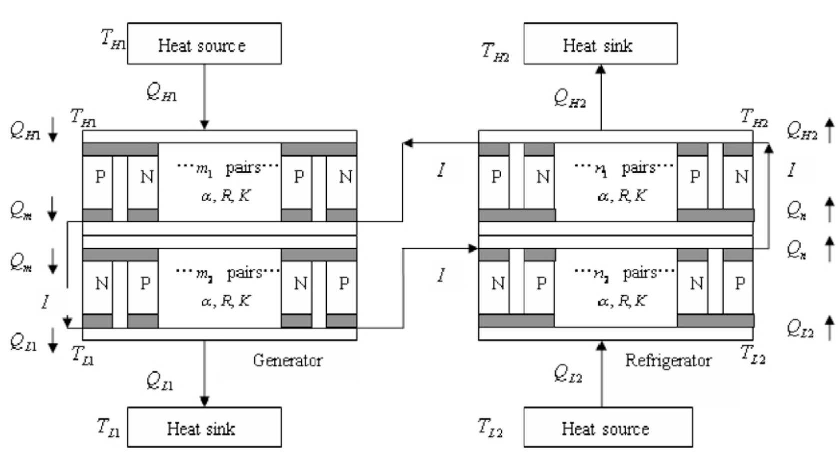

Fig. 1. Schematic diagram of the combined thermoelectric device.

where $\alpha = \alpha_P - \alpha_N$, α_P, and α_N are the Seebeck coefficients of the P- and N-type semiconductor legs for each thermoelectric power generation or refrigerating element.

Combining Eq. (2) with Eq. (3) gives T_m. Substituting it into Eqs. (1) and (4) yields Q_{H1} and Q_{L1}. Combining Eq. (6) with Eq. (7) gives T_n. Substituting it into Eqs. (5) and (8) yields Q_{H2} and Q_{L2}. According to energy balance equation $Q_{H1} + Q_{L2} = Q_{L1} + Q_{H2}$, one can obtain Eq. (A8) that should be satisfied by a stable electrical current, see Appendix A.

For the given parameters α, K, R, T_{H1}, T_{L1}, T_{H2}, T_{L2}, m_1, m_2, n_1, n_2, one can obtain the system stable current I_s (see Appendix A), and then obtain the cooling load and COP of the combined thermoelectric device as follows:

$$Q_{L2} = n_2\{\alpha I_s T_{L2} - K[(0.5n_1 I_s^2 R + 0.5n_1 I_s^2 R + n_1 K T_{H1} + n_2 K T_{H2})/(n_1\alpha I_s - n_2 \alpha I_s + n_1 K + n_2 K) - T_{L2}] - 0.5 I_s^2 R\} \quad (9)$$

$$\varepsilon = Q_{L2}/Q_{H1} = \frac{n_2\{\alpha I_s T_{L2} - K[(0.5n_1 I_s^2 R + 0.5n_1 I_s^2 R + n_1 K T_{H1} + n_2 K T_{H2})/(n_1\alpha I_s - n_2\alpha I_s + n_1 K + n_2 K) - T_{L2}] - 0.5 I_s^2 R\}}{m_1\{\alpha I_s T_{H1} + K[T_{H1} - (0.5m_1 I_s^2 R + 0.5m_2 I_s^2 R + m_1 K T_{H1} + m_2 K T_{L1})/(m_2\alpha I_s - m_1\alpha I_s + m_1 K + m_2 K)] - 0.5 I_s^2 R\}} \quad (10)$$

$\varepsilon_{Q_{L2}}$. In general, the optimum allocations of thermoelectric element pairs and optimum working currents at maximum cooling load are different from these at maximum COP. The maximum cooling load of the combined thermoelectric device, $Q_{L2,\max}$, is directly proportional to the total number of thermoelectric element pairs (see Fig. 2), and the maximum COP, $\varepsilon_{\max}$, is independent of the total number of thermoelectric element pairs. The optimum working currents corresponding to the maximum cooling load, $I_{\mathrm{opt},Q_{L2}}$, and the optimum working currents corresponding to the maximum COP, $I_{\mathrm{opt},\varepsilon}$, are also independent of the total number of thermoelectric element pairs, and $I_{\mathrm{opt},Q_{L2}} > I_{\mathrm{opt},\varepsilon}$ holds. The cooling load at maximum COP, $Q_{L2,\varepsilon}$, increases with the increase of the total number of thermoelectric element pairs, while the COP at maximum cooling load, $\varepsilon_{Q_{L2}}$, is independent of the total number of thermoelectric element pairs.

Although the optimum thermoelectric element allocations among the two thermoelectric generators and the two thermoelectric refrigerators are different for different total number of thermoelectric element pairs and different optimizing objectives, there are some general rules. Three ratios of number of thermoelectric element pairs are defined: total element ratio of generator to refrigerator, $x = m/M$, generator element ratio, $x_1 = m_1/m$, and refrigerator element ratio, $x_2 = n_1/n$. Table 2 lists the results of optimum ratios of number of thermoelectric element pairs. One can see that three optimum ratios of number of thermoelectric element pairs for maximum cooling load, $x_{\mathrm{opt},Q_{L2}}$, $x_{1\mathrm{opt},Q_{L2}}$, and $x_{2\mathrm{opt},Q_{L2}}$, and three optimum ratios of number of thermoelectric element pairs for maximum COP, $x_{\mathrm{opt},\varepsilon}$, $x_{1\mathrm{opt},\varepsilon}$, and $x_{2\mathrm{opt},\varepsilon}$, are independent of the total number of thermoelectric element pairs. If m_1, m_2, n_1, and n_2 in Eqs. (A9)-(A12) are replaced by M, x, x_1, and x_2, one can see that the equation for the stable working electrical current, Eq. (A8) is independent of the total number of thermoelectric element pairs, M. In theory, given x, x_1, and x_2, the total number of thermoelectric element pairs, M does not affect the working electrical current. In practice, the numbers of the thermoelectric element pairs of the two generators and the two refrigerators, m_1, m_2, n_1, and n_2 must be integers, which makes the optimum ratios, x_{opt}, $x_{1\mathrm{opt}}$, and $x_{2\mathrm{opt}}$ vary a little with the total number of thermoelectric element pairs, M. In this example, one can see that $x_{\mathrm{opt},Q_{L2}} \approx 0.64$ and $x_{\mathrm{opt},\varepsilon} \approx 0.51$. That is, $x_{\mathrm{opt},Q_{L2}} > x_{\mathrm{opt},\varepsilon}$, the optimum total element ratio of generator to refrigerator at maximum cooling load is larger than that at maximum COP. This means that the optimum allocation range is $x_{\mathrm{opt},\varepsilon} \leqslant x \leqslant x_{\mathrm{opt},Q_{L2}}$, more thermoelectric elements in the thermo-

3. Numerical examples

Numerical calculations are performed in order to analyze and optimize the performance of the two-stage thermoelectric refrigerator driven by two-stage thermoelectric generator. The total number of thermoelectric element pairs is finite, i.e. $M = m_1 + m_2 + n_1 + n_2$. The optimum thermoelectric element allocations and the optimum working electrical current, $I_{\mathrm{opt},Q_{L2}}$ or $I_{\mathrm{opt},\varepsilon}$ at maximum cooling load, $Q_{L2,\max}$, or maximum COP, $\varepsilon_{\max}$, are searched, respectively. In the calculations, $\alpha = 2.1 \times 10^{-4}$ V/K, $K = 1.6 \times 10^{-2}$ W/K, and $R = 1.2 \times 10^{-3}\ \Omega$ are used [47].

3.1. Effects of total number and allocation of thermoelectric element pairs

Table 1 lists the optimum parameters and device performance with different total number of thermoelectric element pairs. In the calculations, the temperatures of the heat reservoirs are set to be $T_{H1} = 450$ K, $T_{L1} = 300$ K, $T_{H2} = 300$ K, and $T_{L2} = 280$ K. One can see from the table that there exist optimum thermoelectric element allocations among the two thermoelectric generators and the two thermoelectric refrigerators for the fixed total number of thermoelectric element pairs and there exist optimum working currents, $I_{\mathrm{opt},Q_{L2}}$ or $I_{\mathrm{opt},\varepsilon}$, corresponding to maximum cooling load, $Q_{L2,\max}$, or maximum COP, $\varepsilon_{\max}$, respectively. The cooling load at maximum COP is $Q_{L2,\varepsilon}$, and the COP at maximum cooling load is

Table 1
Optimum parameters and performance of the combined thermoelectric device.

M	Optimizing objective	m_1	m_2	n_1	n_2	I_{opt}, Q_{L2} (A)	$Q_{L2,\max}$ (W)	$\varepsilon_{Q_{L2}}$	$I_{\mathrm{opt},\varepsilon}$ (A)	$Q_{L2,}$ (W)	$\varepsilon_{\max}$
40	Q_{L2}	13	13	8	6	8.01	1.68	0.07			
	ε	10	10	11	9				5.72	1.40	0.08
80	Q_{L2}	26	26	16	12	8.01	3.37	0.07			
	ε	20	20	23	17				5.73	2.82	0.08
120	Q_{L2}	38	38	26	18	7.77	5.06	0.07			
	ε	31	31	33	25				5.98	4.42	0.08
160	Q_{L2}	51	51	34	24	7.83	6.75	0.07			
	ε	41	41	45	33				5.92	5.83	0.08
200	Q_{L2}	64	64	42	30	7.87	8.44	0.07			
	ε	51	51	56	42				5.88	7.24	0.08
400	Q_{L2}	127	127	85	61	7.79	16.88	0.07			
	ε	102	102	112	84				5.88	14.48	0.08

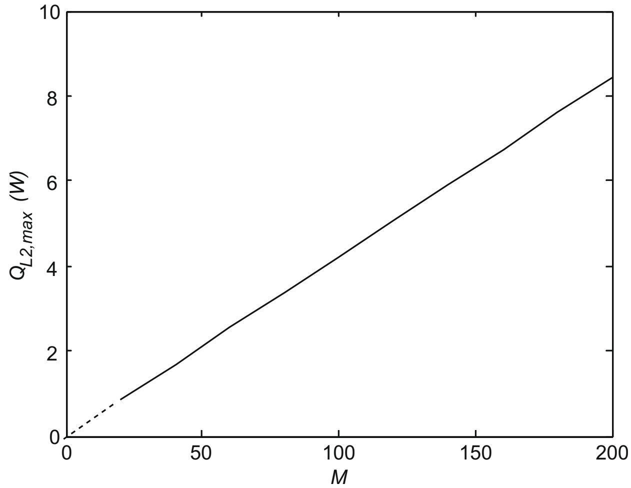

Fig. 2. Maximum cooling load versus total number of thermoelectric element pairs.

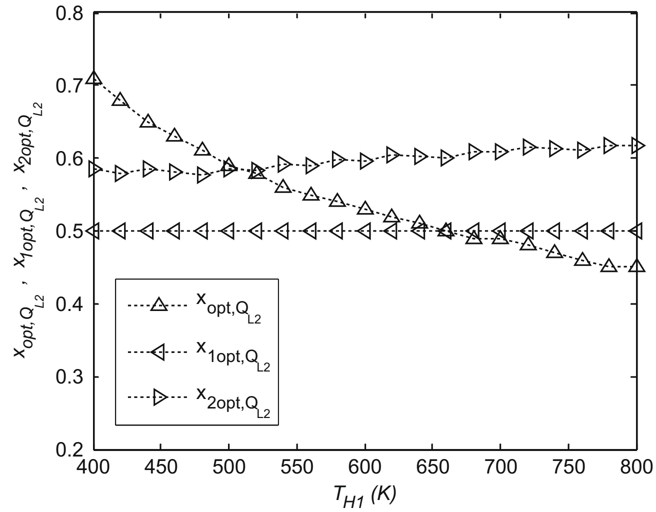

Fig. 3. Optimum parameters at maximum cooling load versus the heat source temperature of the two-stage generator.

electric generators can obtain larger cooling load, and more thermoelectric elements in the thermoelectric refrigerators can obtain larger COP. For the optimum total element ratio of generator to refrigerator, either for maximum cooling load or for maximum COP, the optimum generator element ratios are always $x_{1\mathrm{opt},Q_{L2}} = x_{1\mathrm{opt},\varepsilon} = 0.5$, i.e. the equal partition of number of thermoelectric element between the two thermoelectric generators will lead to both maximum cooling load and maximum COP. While the optimum refrigerator element ratios, $x_{2\mathrm{opt},Q_{L2}}$ and $x_{2\mathrm{opt},\varepsilon}$ are always larger than 0.5, they are also almost the same.

3.2. Effects of heat source temperature of the two-stage thermoelectric generator

The effects of the heat source temperature of the two-stage thermoelectric generator on the optimum parameters and the optimum performance of the combined thermoelectric device are analyzed. In the calculations, $T_{L1} = 300$ K, $T_{H2} = 300$ K, $T_{L2} = 280$ K, and $M = 200$ are set. Figs. 3 and 4 show the characteristic that three optimum ratios of number of thermoelectric element pairs for maximum cooling load ($x_{\mathrm{opt},Q_{L2}}$, $x_{1\mathrm{opt},Q_{L2}}$, and $x_{2\mathrm{opt},Q_{L2}}$) versus the heat source temperature of the two-stage thermoelectric generator and three optimum ratios of number of thermoelectric element pairs for maximum COP ($x_{\mathrm{opt},\varepsilon}$, $x_{1\mathrm{opt},\varepsilon}$, and $x_{2\mathrm{opt},\varepsilon}$) versus the heat source temperature of the two-stage thermoelectric generator, respectively. The corresponding maximum cooling load and maximum COP versus the heat source temperature of the two-stage thermoelectric generator are shown by solid lines in Figs. 5 and 6, respectively. For comparisons, the cooling load of the combined thermoelectric device with $m_1 = 53$, $m_2 = 53$, $n_1 = 56$, and $n_2 = 38$, which are the optimum numbers of the thermoelectric element pairs of the combined thermoelectric device for maximum cooling load objective with $T_{H1} = 600$ K, is shown by dotted line in Fig. 5,

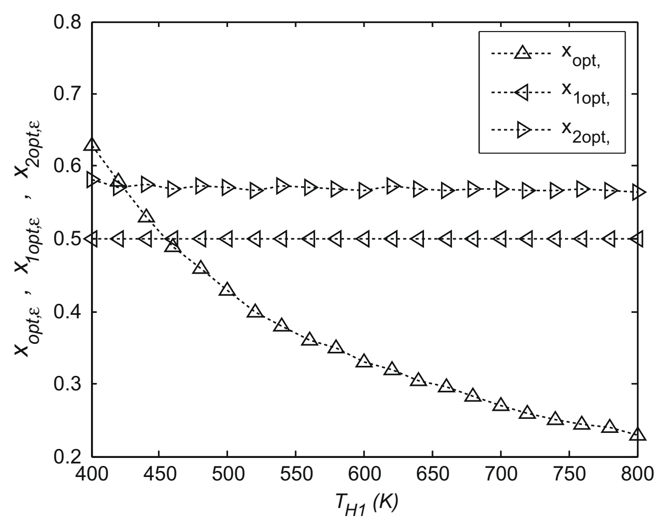

Fig. 4. Optimum parameters at maximum COP versus the heat source temperature of the two-stage generator.

Table 2
Optimum ratios of number of thermoelectric element pairs.

M	Optimizing objective	m_1	m_2	n_1	n_2	x_{opt}, Q_{L2}	$x_{1\mathrm{opt},Q_{L2}}$	$x_{2\mathrm{opt},Q_{L2}}$	$x_{2\mathrm{opt}}, Q_{L2}$	$x_{2\mathrm{opt},\varepsilon}$	$x_{2\mathrm{opt},\varepsilon}$
40	Q_{L2}	13	13	8	6	0.65	0.5	0.57			0
	ε	10	10	11	9				0.50	0.50	0.55
80	Q_{L2}	26	26	16	12	0.65	0.5	0.57			
	ε	20	20	23	17				0.50	0.50	0.58
120	Q_{L2}	38	38	26	18	0.63	0.	0.59			
	ε	31	31	33	25				0.52	0.50	0.57
160	Q_{L2}	51	51	34	24	0.64	0.5	0.59			
	ε	41	41	45	33				0.51	0.50	0.58
200	Q_{L2}	64	64	42	30	0.64	0.5	0.58			
	ε	51	51	56	42				0.51	0.50	0.57
400	Q_{L2}	127	127	85	61	0.64	0.5	0.58			
	ε	102	102	112	84				0.51	0.50	0.57

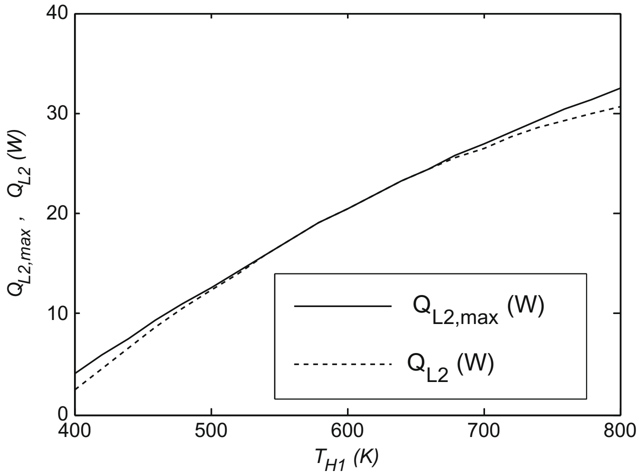

Fig. 5. Maximum cooling load versus the heat source temperature of the two-stage generator.

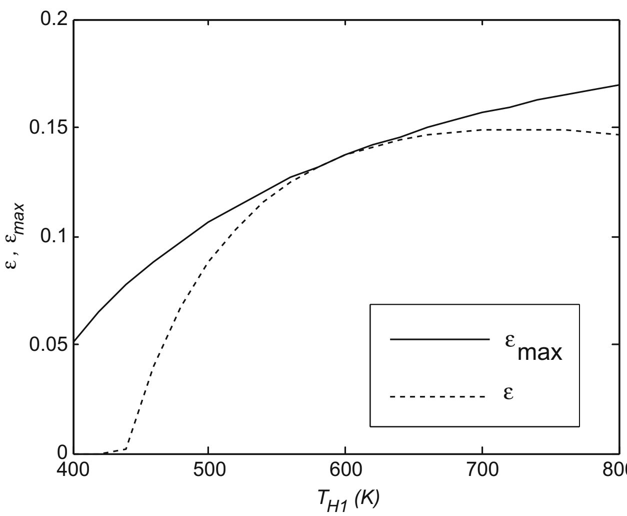

Fig. 6. Maximum COP versus the heat source temperature of the two-stage generator.

and the COP of the combined thermoelectric device with $m_1 = 33$, $m_2 = 33$, $n_1 = 76$, and $n_2 = 58$, which are the optimum numbers of the thermoelectric element pairs of the combined thermoelectric device for maximum COP objective with $T_{H1} = 600$ K, is shown by dotted line in Fig. 6. The corresponding optimum working electrical currents, $I_{Q_{L2}}$, for maximum cooling load and I_ε, for maximum COP versus the heat source temperature of the two-stage thermoelectric generator are shown in Fig. 7.

One can see that the optimum total element ratio of generator to refrigerator $x_{\mathrm{opt},Q_{L2}}$ and $x_{\mathrm{opt},\varepsilon}$ decrease with the increase in the heat source temperature of the two-stage thermoelectric generator. This means that more thermoelectric element pairs must be allocated to the refrigerator in order to obtain maximum cooling load or maximum COP when the heat source temperature of the two-stage thermoelectric generator increases. The selection range of the total element ratio of generator to refrigerator should be $x_\varepsilon \leqslant x \leqslant x_{Q_{L2}}$. More thermoelectric element pairs in the thermoelectric generator can obtain larger cooling load, and more thermoelectric element pairs in the thermoelectric refrigerator can obtain larger COP.

The optimum generator element ratio at maximum cooling load, $x_{1\mathrm{opt},Q_{L2}}$ and the optimum generator element ratio at maximum COP, $x_{1\mathrm{opt},\varepsilon}$, are always 0.5 for different heat source temperature of the two-stage thermoelectric generator.

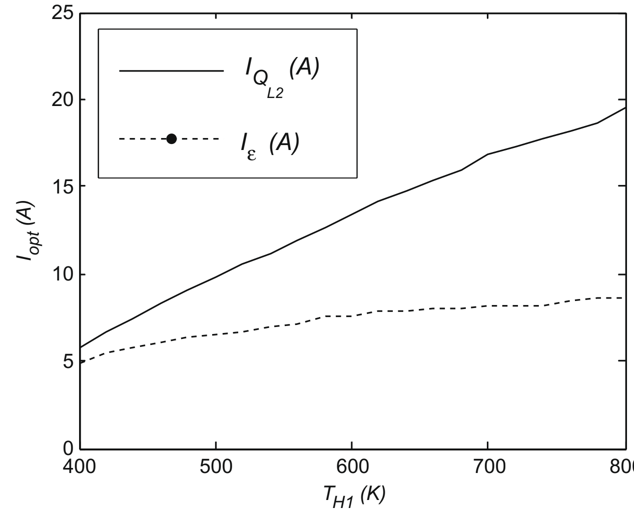

Fig. 7. Optimum working currents versus the heat source temperature of the two-stage generator.

The optimum refrigerator element ratios, $x_{2\mathrm{opt},Q_{L2}}$ and $x_{2\mathrm{opt},\varepsilon}$, change little with the increase of the heat source temperature of the two-stage thermoelectric generator.

Both the maximum cooling load and the maximum COP increase monotonically with the increase of the heat source temperature of the two-stage thermoelectric generator (See solid lines in Figs. 5 and 6). If the allocations of the thermoelectric element pairs among the two thermoelectric generators and the two thermoelectric refrigerators do not meet the needs of optimum allocations for maximum cooling load or maximum COP, both cooling load and COP are less than the maximum ones.

Both the optimum working currents $I_{Q_{L2}}$ and I_ε increase with the increase of the heat source temperature of the two-stage thermoelectric generator, and $I_{Q_{L2}} \geqslant I_\varepsilon$ holds. The difference between $I_{Q_{L2}}$ and I_ε increase with the increase of T_{H1} and if $T_{H1} \to T_{L1} = 300$ K, $I_{Q_{L2}} - I_\varepsilon = T_d \to 0$. The optimum working currents for both heating load and COP are $I_\varepsilon \leqslant I_{\mathrm{opt}} \leqslant I_{Q_{L2}}$.

3.3. Effects of heat source temperature of the two-stage thermoelectric refrigerator

The effects of the heat source (cooling space) temperature of the two-stage thermoelectric refrigerator on the optimum parameters and the optimum performance of the combined thermoelectric device are analyzed. In the calculations, $T_{H1} = 800$ K, $T_{L1} = 300$ K, $T_{H2} = 300$ K, and $M = 200$ are set. Figs. 8 and 9 show the characteristic that three optimum ratios of number of thermoelectric element pairs for maximum cooling load ($x_{\mathrm{opt},Q_{L2}}$, $x_{1\mathrm{opt},Q_{L2}}$, and $x_{2\mathrm{opt},Q_{L2}}$) versus the heat source temperature of the two-stage thermoelectric refrigerator and three optimum ratios of number of thermoelectric element pairs for maximum COP ($x_{\mathrm{opt},\varepsilon}$, $x_{1\mathrm{opt},\varepsilon}$, and $x_{2\mathrm{opt},\varepsilon}$) versus the heat source temperature of the two-stage thermoelectric refrigerator, respectively. The corresponding maximum cooling load and maximum COP versus the heat source temperature of the two-stage thermoelectric refrigerator are shown by solid lines in Figs. 10 and 11, respectively. For comparisons, the cooling load of the combined thermoelectric device with $m_1 = 45$, $m_2 = 45$, $n_1 = 68$, and $n_2 = 42$, which are the optimum numbers of the thermoelectric element pairs of the combined thermoelectric device for maximum cooling load objective with $T_{L2} = 280$ K, is shown by dotted line in Fig. 10, and the COP of the combined ther-

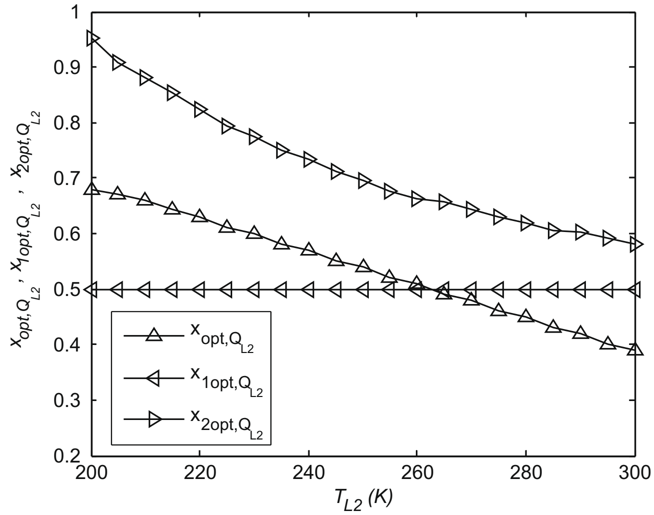

Fig. 8. Optimum parameters at maximum cooling load versus the heat source temperature of the two-stage refrigerator.

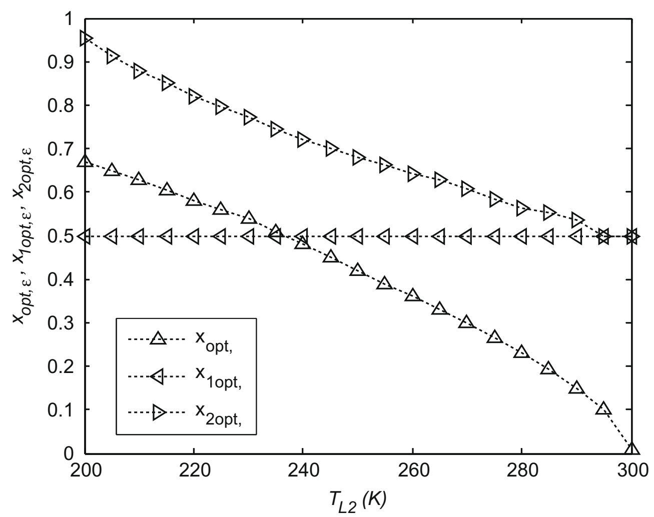

Fig. 9. Optimum parameters at maximum COP versus the heat source temperature of the two-stage refrigerator.

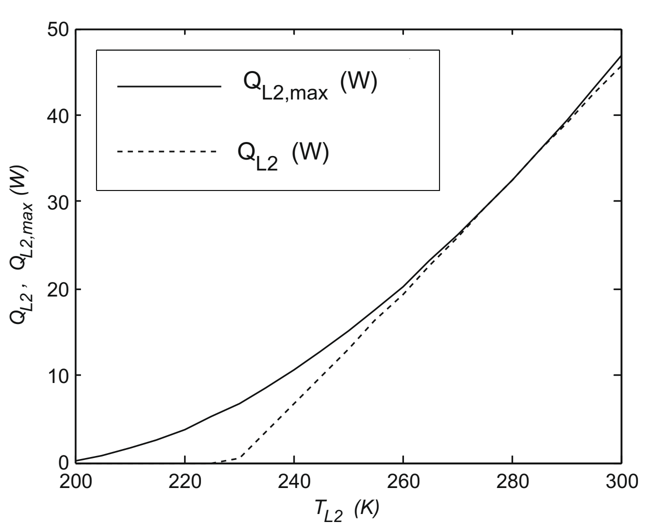

Fig. 10. Maximum cooling load versus the heat source temperature of the two-stage refrigerator.

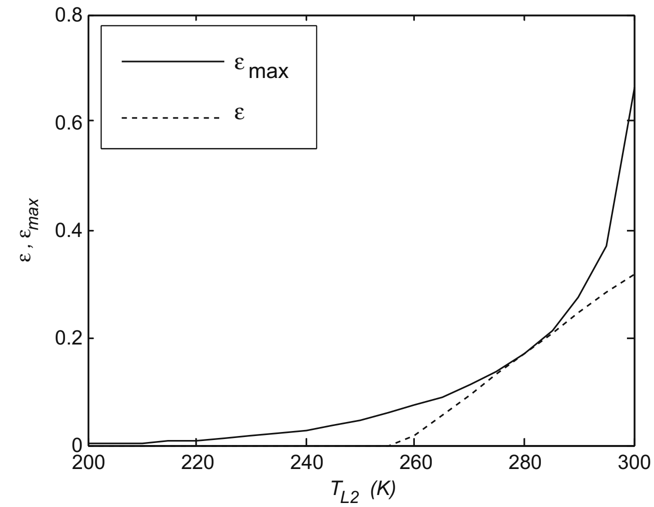

Fig. 11. Maximum COP versus the heat source temperature of the two-stage refrigerator.

moelectric device with $m_1 = 20$, $m_2 = 26$, $n_1 = 87$, and $n_2 = 67$, which are the optimum numbers of the thermoelectric element pairs of the combined thermoelectric device for maximum COP objective with $T_{L2} = 280$ K, is shown by dotted line in Fig. 11. The corresponding optimum working currents, $I_{Q_{L2}}$ and I_{ε} versus the heat source temperature of the two-stage thermoelectric refrigerator are shown in Fig. 12.

One can see that the optimum total element ratio of generator to refrigerator $x_{opt,Q_{L2}}$ and $x_{opt,\varepsilon}$ decrease with the increase of the heat source temperature of the two-stage thermoelectric refrigerator. This means more thermoelectric element pairs must be allocated to the refrigerator in order to obtain maximum cooling load or maximum COP when the heat source temperature of the two-stage thermoelectric refrigerator increases.

Both the optimum generator element ratio $x_{1opt,Q_{L2}}$ at maximum cooling load and the optimum generator element ratio $x_{1opt,\varepsilon}$ at maximum COP are always 0.5 for different heat source temperature of the two-stage thermoelectric refrigerator.

The optimum refrigerator element ratios $x_{2opt,Q_{L2}}$ and $x_{2opt,\varepsilon}$ decrease with the increase of the heat source temperature of the

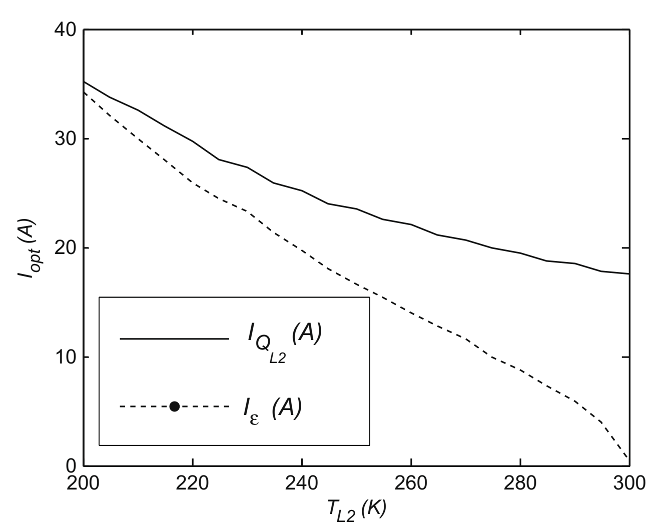

Fig. 12. Optimum working currents versus the heat source temperature of the two-stage refrigerator.

two-stage thermoelectric refrigerator. This means that more thermoelectric element pairs must be allocated to the bottom stage of the refrigerator in order to obtain maximum cooling load or maximum COP when the heat source temperature of the two-stage thermoelectric refrigerator increases.

Both the maximum cooling load and the maximum COP increase rapidly with the increase of the heat source temperature of the two-stage thermoelectric refrigerator. If the allocations of the thermoelectric element pairs among the two thermoelectric generators and the two thermoelectric refrigerators do not meet the needs of optimum allocations for maximum cooling load or maximum COP, both the cooling load and the COP are less than the maximum ones.

Both the optimum working currents $I_{Q_{L2}}$ and I_ε decrease with the increase of the heat source temperature of the two-stage thermoelectric refrigerator, and $I_{Q_{L2}} \geqslant I_\varepsilon$ holds. The difference between $I_{Q_{L2}}$ and I_ε increases with the increase of T_{L2} and if $T_{H1} \to 200$ K (that is the minimum cooling temperature in this condition), $I_{Q_{L2}} - I_\varepsilon = T_d \to 0$. The optimum working currents for both heating load and COP are $I_\varepsilon \leqslant I_{\text{opt}} \leqslant I_{Q_{L2}}$.

4. Conclusion

A new configuration of combined thermoelectric device, two-stage thermoelectric refrigerator driven by two-stage thermoelectric generator, is proposed in this paper. The thermodynamic model of the combined device is built by using non-equilibrium thermodynamic theory. Three analytical formulae for the stable working electrical current, the cooling load versus the working electrical current, and the COP versus the working electrical current of the combined device are derived. The performance optimization of the combined thermoelectric device is performed by searching the allocations of the thermoelectric element pairs among the two thermoelectric generators and the two thermoelectric refrigerators. There has been no investigation concerning the performance analysis and optimization for such combined thermoelectric device published in the open literature. The influences of the heat source temperature of the two-stage thermoelectric generator and the heat source (cooling space) temperature of the two-stage thermoelectric refrigerator on the optimal performance of the combined thermoelectric device are analyzed. All the parameters should be considered in the design and application of practical combined thermoelectric devices in order to obtain the maximum economy benefit.

The results show that there exist optimum thermoelectric element allocations among the two thermoelectric generators and the two thermoelectric refrigerators for the fixed total number of thermoelectric element pairs and there exist the optimum working currents corresponding to maximum cooling load or maximum COP, respectively. In general, the optimum allocations of thermoelectric element pairs and optimum working currents at maximum cooling load are different from these at maximum COP. The optimum working currents corresponding to the maximum cooling load is larger than that corresponding to the maximum COP. The cooling load at maximum COP increases with the increase of the total number of thermoelectric element pairs, while the COP at maximum cooling load is independent of the total number of thermoelectric element pairs. Three optimum ratios of number of thermoelectric element pairs for maximum cooling load and three optimum ratios of number of thermoelectric element pairs for maximum COP are also independent of the total number of thermoelectric element pairs.

The results obtained herein may provide guidelines for the design and application of practical combined thermoelectric devices.

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Appendix A. A stable electrical current

Combining Eq. (2) with Eq. (3) gives

$$T_m = \frac{(m_1+m_2)I^2R/2 + m_1KT_{H1} + m_2KT_{H2}}{\alpha I(m_2-m_1) + K(m_1+m_2)} \tag{A1}$$

Combining Eq. (6) with Eq. (7) gives

$$T_n = \frac{(n_1+n_2)I^2R/2 + n_1KT_{H1} + n_2KT_{H2}}{\alpha I(n_1-n_2) + K(n_1+n_2)} \tag{A2}$$

Substituting Eq. (A1) into Eqs. (1) and (4) yields

$$\begin{aligned} Q_{H1} = m_1\{&\alpha IT_{H1} + K[T_{H1} - (0.5m_1I^2R + 0.5m_2I^2R + m_1KT_{H1} \\ &+ m_2KT_{L1})/(m_2\alpha I - m_1\alpha I + m_1K + m_2K)] - 0.5I^2R\} \end{aligned} \tag{A3}$$

$$\begin{aligned} Q_{L1} = m_2\{&\alpha IT_{L1} + K[(0.5m_1I^2R + 0.5m_2I^2R + m_1KT_{H1} \\ &+ m_2KT_{L1})/(m_2\alpha I - m_1\alpha I + m_1K + m_2K) - T_{L1}] \\ &+ 0.5I^2R\} \end{aligned} \tag{A4}$$

Substituting Eq. (A2) into Eqs. (5) and (8) yields

$$\begin{aligned} Q_{H2} = n_1\{&\alpha IT_{H2} - K[T_{H2} - (0.5n_1I^2R + 0.5n_1I^2R + n_1KT_{H1} \\ &+ n_2KT_{H2})/(n_1\alpha I - n_2\alpha I + n_1K + n_2K)] + 0.5I^2R\} \end{aligned} \tag{A5}$$

$$\begin{aligned} Q_{L2} = n_2\{&\alpha IT_{L2} - K[(0.5n_1I^2R + 0.5n_1I^2R + n_1KT_{H1} \\ &+ n_2KT_{H2})/(n_1\alpha I - n_2\alpha I + n_1K + n_2K) - T_{L2}] - 0.5I^2R\} \end{aligned} \tag{A6}$$

The overall system is a closed loop circuit, and heat flow of the system is in balance, one has

$$Q_{H1} + Q_{L2} = Q_{L1} + Q_{H2} \tag{A7}$$

Substituting Eqs. (A3)-(A6) into Eq. (A7) and re-arranging the results yields the equation that should be satisfied by a stable electrical current I

$$A_3I^3 + A_2I^2 + A_1I + A_0 = 0 \tag{A8}$$

where

$$A_3 = \alpha^2R(m_1-m_2)(n_2-n_1)(m_1+m_2+n_1+n_2) \tag{A9}$$

$$\begin{aligned} A_2 = \alpha[&\alpha^2(n_1-n_2)(m_1-m_2)(m_1T_{H1} + n_2T_{L2} - n_1T_{H2} \\ &- m_2T_{L1}) + 0.5RK(4n_2n_1m_1 - n_2^2m_2 - 3m_2^2n_1 + n_2m_2^2 \\ &- m_1^2n1 + 3m_1^2n_2 + n_1^2m_1 - 4n_2n_1m_2 + 3n_2^2m_1 \\ &+ 4m_2m_1n_2 - 4m_1m_2n_1 - 3n_1^2m_2)] \end{aligned} \tag{A10}$$

$$\begin{aligned} A_1 = 2\alpha^2K(&n_1n_2m_2T_{H2} - 3m_1m_2n_1T_{H1} + m_1m_2n_2T_{H1} \\ &- 3m_2m_1n_2T_{L1} + m_2m_1n_1T_{L1} + m_1^2n_1T_{H1} + m_2^2n_2T_{L1} \\ &+ m_2^2n_1T_{L1} - 3n_1n_2m_1T_{H2} + n_2^2m_1T_{L2} + n_1^2m_1T_{H2} \\ &+ m_1^2n_2T_{H1} + n_2n_1m_1T_{L2} - 3n_2n_1m_2T_{L2} + n_2^2m_2T_{L2} \\ &+ n_1^2m_2T_{H2}) - 2RK^2(n_1+n_2)(m_1+m_2)(m_1+m_2+n_1 \\ &+ n_2) \end{aligned} \tag{A11}$$

$$A_0 = 4\alpha K^2(-m_1T_{H1}m_2n_2 + n_1T_{H2}n_2m_1 + n_1T_{H2}n_2m_2 - m_1T_{H1}m_2n_1 + m_2T_{L1}m_1n_1 + m_2T_{L1}m_1n_2 - n_2T_{L2}n_1m_1 - n_2T_{L2}n_1m_2) \quad (A12)$$

The device practical electrical current (the solution of Eq. (A8)) is dependent on the values of m_1, m_2, n_1 and n_2.

When $m_1 = m_2$ or $n_1 = n_2$, then one has that $A_3 = A_2 = 0$ and the practical electrical current I_s is

$$I_s = I_0 = -\frac{A_0}{A_1} \quad (A13)$$

When $m_1 \neq m_2$ and $n_1 \neq n_2$, the solutions of Eq. (A8) are

$$I_1 = B_1 + B_2 + \frac{\sqrt{3}}{2}B_3 i \quad (A14)$$

$$I_2 = B_1 + B_2 - \frac{\sqrt{3}}{2}B_3 i \quad (A15)$$

$$I_3 = B_2 + B_3 \quad (A16)$$

where

$$B_1 = -\frac{C^{1/3}}{12A_3} + \frac{3A_1A_3 - A_2^2}{3A_3C^{1/3}} \quad (A17)$$

$$B_2 = -\frac{1}{3}\frac{A_2}{A_3} \quad (A18)$$

$$B_3 = \frac{C^{1/3}}{6A_3} + \frac{6A_1A_3 - 2A_2^2}{3A_3C^{1/3}} \quad (A19)$$

and

$$C = 36A_1A_2A_3 - 108A_0A_3^2 - 8A_2^3 + 12\sqrt{3}A_3(4A_1^3A_3 - A_1^2A_2^2 - 18A_0A_1A_2A_3 + 27A_0^2A_3^2 + 4A_0A_2^3)^{1/2} \quad (A20)$$

Analysis shows that the practical stable electrical current solution I_s must satisfy the following rules:

(1) When $m_1 < m_2$ and $n_1 > n_2$, the solution is Eq. (A14): $I_s = I_1 = B_1 + B_2 + \frac{\sqrt{3}}{2}B_3 i$.
(2) When $m_1 > m_2$ and $n_1 < n_2$, the solution is Eq. (A14): $I_s = I_1 = B_1 + B_2 + \frac{\sqrt{3}}{2}B_3 i$.
(3) When $m_1 < m_2$ and $n_1 < n_2$, the solution is Eq. (A15): $I_s = I_2 = B_1 + B_2 - \frac{\sqrt{3}}{2}B_3 i$.
(4) When $m_1 > m_2$ and $n_1 > n_2$ the solution is Eq. (A15): $I_s = I_2 = B_1 + B_2 - \frac{\sqrt{3}}{2}B_3 i$.
(5) When $m_1 = m_2$ or $n_1 = n_2$, the solution is Eq. (A13): $I_s = I_0 = -\frac{A_0}{A_1}$.

For the given parameters α, K, R, T_{H1}, T_{L1}, T_{H2}, T_{L2}, m_1, m_2, n_1, n_2, one can obtain the system stable current I_s, and then obtain the cooling load and COP of the combined thermoelectric device, see Eqs. (9) and (10).

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